Shaping the Kuiper Belt Size Spectrum by Shattering Large but Strengthless Bodies

Margaret Pan and Re'em Sari

130-33 Caltech, Pasadena, CA 91125

ABSTRACT

The observed size distribution of Kuiper belt objects (KBOs)—small icy and rocky solar system bodies orbiting beyond Neptune—is well described by a power law at large KBO sizes. However, recent work by Bernstein et al. (2003) indicates that the size spectrum breaks and becomes shallower for KBOs smaller than about 70 km in size. Here we show that we expect such a break at KBO radius ∼40 km since destructive collisions are frequent for smaller KBOs. Specifically, we assume that KBOs are rubble piles with low material strength rather than solid monoliths. This gives a power-law slope $q \simeq 3$ where the number $N(r)$ of KBOs larger than a size r is given by $N(r) \propto r^{1-q}$; the break location follows from this slope through a self-consistent calculation. The existence of this break, the break's location, and the power-law slope we expect below the break are consistent with the findings of Bernstein et al. (2003). The agreement with observations indicates that KBOs are effectively strengthless rubble piles.

1. Introduction

The Kuiper belt, a population of small bodies moving beyond the giant planets, was discovered when its first member was found in 1992 (Jewitt & Luu 1993). As of late 2003, ∼800 KBOs have been discovered. Due to KBOs' faintness, however, the size distribution of KBOs is well determined observationally only for bodies larger than ∼100 km (Trujillo et al. 2001, Gladman et al. 1998, Chiang & Brown 1999); their size spectrum is consistent with a power law $N(r) \propto r^{-4}$ (Bernstein et al. 2003). Numerical studies concluded that the differential size spectrum below ∼100 km should follow a power law with the slightly shallower $N \propto r^{-2.5}$ due to the effects of destructive collisions (Farinella & Davis 1996, Davis & Farinella 1997, Kenyon 2002). The results seemed consistent with loose observational constraints available on the number of ∼20 km and ∼2 km KBOs (Cochran et al. 1995, Holman & Wisdom 1993).

In this context, the deficit in small KBOs observed by Bernstein et al. (2003) was a surprise. Using the Advanced Camera for Surveys recently installed on the Hubble Space Telescope, they found just 3 KBOs of size ∼25–45 km where they expected ∼85 such bodies based on an extrapolation of the accepted best-fit large-KBO spectrum at the time (Trujillo et al. 2001). While this observed decrement of more than an order of magnitude in the number of small KBOs clearly indicates a break between 45 and 100 km, the exact break position and slope below the break may well be refined by future data on small KBOs. Still, the results of Bernstein et al. (2003) are inconsistent with the previously expected small-end spectrum $N \propto R^{-2.5}$, or $q = 3.5$, at better than 95% confidence.

This paper describes a simple self-consistent analytic calculation of the break location and the slope below the break. Note that using the $N(r) \propto r^{-4}$ size spectrum obtained by Bernstein et al. (2003) for large KBOs, we can estimate the size below which collisions between equal size bodies should be frequent to be ~ 1 km—well below the observed break location. However, this estimate needs two modifications. First, due to the large velocity dispersion in the Kuiper belt, small bodies can shatter much larger objects. Since there are more small than large bodies, destructive collisions will occur frequently even for objects much larger than 1 km. Second, when collisions are important, they reduce the number of small bodies; this in turn decreases the frequency of collisions. Therefore, calculations of the effects of collisions and the size below which collisions are important must be done in a self-consistent manner.

2. Slope of the steady-state distribution

In order to find the break location self-consistently, we first calculate the power-law slope q for a collisional population of bodies. We assume a group of bodies with isotropic velocity dispersion v in which the differential number of bodies of radius r is given by a power law $dN(r)/dr \propto r^{-q}$. If we assume that the population is in a steady state and that mass is conserved in the collision process, the total mass of bodies destroyed per unit time in a logarithmic interval in radius must be independent of size. We can use this condition to determine q.

We assume that the main channel for mass destruction is the shattering of larger 'targets' by smaller 'bullets' (Fig. 1). Under this condition,

$$\rho r^3 \cdot N(r) \cdot \frac{N(r_B)}{V} \cdot r^2 \cdot v = \text{constant} \quad . \tag{1}$$

Here ρ is the internal density of each body and $r_B(r)$ is the size of the smallest bullet which, on impact, can shatter a target of radius r. V is the volume occupied by all the bodies;

their velocity dispersion and therefore their distribution within V are assumed independent of body size. When supplemented by a relation between the size of the bullet and that of the target, Eq. 1 dictates the size spectrum q.

This very simple formalism based on conservation of mass captures the essence of Dohnanyi's (1969) more elaborate pioneering treatment. Based on laboratory experiments which involved solid bodies dominated by material strength, Dohnanyi chose $r_B \propto r$. When $r_B \propto r$ and $N(r) \propto r^{1-q}$ are inserted into Eq. 1, we retrieve the $q = 7/2$ of Dohnanyi and several subsequent authors (for example, Williams & Wetherill 1994, Tanaka et al. 1996). This slope is much steeper than the best-fit small-end $q = 2.3$ found by Bernstein et al. (2003), who rule out $q = 7/2$ at better than 95% confidence.

Indeed, work on the structure of small solar system bodies suggests that many of them are gravitationally bound rubble piles rather than solid monoliths. Based on oscillating lightcurves of the large KBO (20000) Varuna (radius $R > 100$ km), Jewitt & Sheppard (2002) find that this bodies has density ~ 1 g cm^{-3} and is therefore unlikely to be solid. However, other effects may also be responsible for the lightcurve shape (see, for example, Goldreich et al. 2004). The rotation statistics of much smaller bodies ($R \sim 10$ km) in the more easily observed region between the asteroid belt and the sun also suggest that small bodies in the solar system are rubble piles rather than monoliths. That no small asteroids are observed to rotate faster than their breakup speed suggests that those which were spun up beyond breakup simply broke apart (Harris 1996); this in turn suggests that these asteroids have no tensile strength. A study including 26 small near-earth asteroids came to similar conclusions about asteroid internal structure (Pravec et al. 1998). The most detailed probe available of the structure of small KBOs is research on short-period comets, kilometer-sized bodies which are thought to have originated in the Kuiper belt. Work on the breakup and impact of comet Shoemaker-Levy 9, thought to be 1–2 km in size, indicates that its strength before breakup was ~ 60 dyn cm^{-2} or less (Asphaug & Benz 1996); a body like Shoemaker-Levy 9 would have material strength energy at most about ten times less than its gravitational energy. These indications motivate an investigation of the influence of negligible material strength on the fragmentation spectrum.

We might therefore replace the $r_B \propto r$ destruction criterion used by Dohnanyi with the requirement that the kinetic energy of the bullet be equal to the total gravitational energy of the target:

$$\rho r_B^3 v^2 \sim \rho r^3 v_{\rm esc}^2 \tag{2}$$

where $v_{\rm esc} \propto r$ is the escape velocity from a target of size r, and, again, v is the bodies'

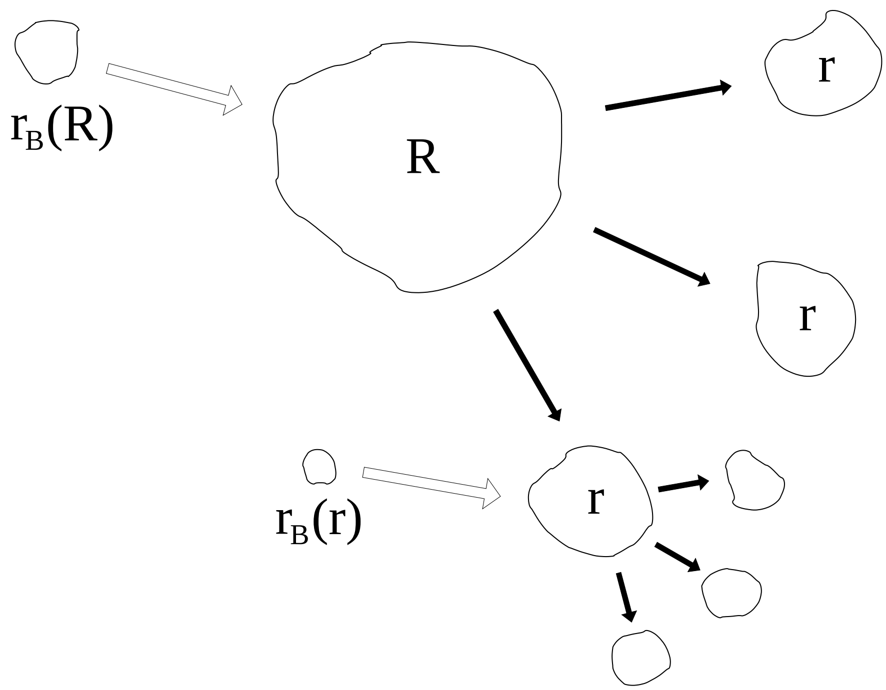

Fig. 1.— Schematic of the collisional cascade: bullets of size $r_B(R)$ shatter targets of typical size R; these targets break into new targets of size r, which are in turn shattered by bullets of size $r_B(r)$; and so on. Since mass is conserved in collisions, the mass destruction rate of bodies of size R is the mass creation rate of bodies of size r. Steady state then requires that the rate of mass destruction be independent of body size.

constant velocity dispersion. Then

$$r_B(r) \sim \left(\frac{G\rho}{v^2}\right)^{1/3} r^{5/3} \sim r_{\rm eq}^{-2/3} r^{5/3} \quad , \quad r_{\rm eq} \sim \frac{v}{\sqrt{G\rho}} \quad . \tag{3}$$

Physically, $r_{\rm eq}$ is the size of a body whose escape velocity equals the velocity dispersion of the system. When density $\rho \sim 1$ g cm^{-3} and the Kuiper Belt's current velocity dispersion $v \sim 1$ km s^{-1} are used, $r_{\rm eq} \sim 10^3$ km $\sim$ the radius of Pluto. Equivalently, a target of size r_{eq}, or roughly Pluto's size, would require a bullet of equal mass to shatter it. Then a body smaller than Pluto—that is, virtually any KBO—can be shattered by bullets smaller than itself. When we substitute Eq. 3, or essentially the proportionality $r_B \propto r^{5/3}$, into Eq. 1, we get the power-law slope

$$q = 23/8 \quad . \tag{4}$$

Recently, O'Brien & Greenberg (2003) extended Dohnanyi's treatment to other destruction conditions where r_B scales as an arbitrary power of r; they show that q is a simple function of this power so that a range of q values can be obtained from a calculation like Dohnanyi's. The simple argument we express in Eq. 1 reproduces their analytic results for q. Eq. 3 is a special cases of their general power law which is clearly motivated by energy considerations and which leads to the spectrum given by Eq. 4.

3. Realistic destruction criteria

The destruction criterion just discussed neglects any energy loss during the impact process. It is then a lower limit on the energy needed to shatter and disperse a given target. Indeed, numerical simulations and dimensional analysis of impact events find that in the 'gravity regime', or target size range where gravity dominates material strength, the impact energy needed to shatter a given target lies well above the level indicated by Eq. 3 (Housen & Holsapple 1990, Holsapple 1994, Love & Ahrens 1996, Melosh & Ryan 1997, Benz & Asphaug 1999). Further, the $r_B(r)$ scalings indicated by these studies[1] are consistently shallower than the one in Eq. 3. With $r_B \propto r^{\alpha}$, they give $1.37 \le \alpha \le 1.57$ rather than the $\alpha = 5/3$ in Eq. 3.

Upon insertion into Eq. 1, the $r_B(r)$ scalings above give $2.95 < q < 3.11$. These values indicate a spectrum between the one given by Eq. 4 and Dohnanyi's $q = 3.5$. This range in q is consistent with the best-fit slope $q = 2.8 \pm 0.6$ (95% confidence) derived by Bernstein et al. (2003) below the break for the classical Kuiper belt and with the best-fit $q = 2.3^{+0.9}_{-1.1}$ (bounds of 68% confidence contour) slope they find for the entire Kuiper belt. The value

[1]Again, we assume a constant velocity dispersion for the collisional population.

for the entire belt may be skewed downwards by the scattered Kuiper belt data, which include too few faint objects for the scattered belt's small-end slope to be well determined. The observed KBO spectrum is thus consistent with the assumption that gravity dominates material strength in KBOs of size near $r_{\rm break}$.

That the simulations give $r_B(r)$ scalings shallower than that of Eq. 3 implies that the energy lost in a catastrophic collision depends on the bullet/target size ratio. As has previously been noted (see, for example, Melosh & Ryan 1997), we would expect energy loss in the impact of a small bullet on a much larger target. Initially the bullet would transfer most of its energy to a volume the size of itself at the impact site; much of this energy would escape from the site via a small amount of fast ejecta, though some would propagate through the target as a shock.

Somewhat more quantitatively, we can think of a collision between a very small bullet and a large target as a point explosion on the planar surface between a vacuum and a half-infinite space filled with matter. The analogous explosion in a uniform infinite material leads to the Sedov-Taylor blast wave, a self-similar solution of the first type in which total energy is conserved as the spherical shock propagates (Sedov 1946, Taylor 1950). By contrast, a point explosion in a half-infinite space is a self-similar solution of the second type (Zel'dovich & Raizer 1967); the shock moving into the half-space must lose energy as some of the shocked material flows into the vacuum. Also, the nonzero pressure increases the momentum in the shock. So as the shock propagates, its velocity should fall off faster than it would have given conservation of energy but slower than it would have in the case of momentum conservation.

We can use these considerations to constrain $r_B(r)$ scalings for catastrophic collisions. We assume that a given target is destroyed if the velocity of the shock wave when it reaches the antipode of the impact site exceeds the escape velocity (see, for example, Melosh et al. 1994). Let the shock velocity decay as $v_{\rm shock} \propto x^{-\beta}$ where x is the distance traveled by the shock. If the energy in the shock were conserved, we would expect $\beta = 3/2$ from dimensional analysis; if the momentum were conserved, we would expect $\beta = 3$. Then the actual point explosion solution must have $3/2 < \beta < 3$. The criterion for target destruction is

$$\rho r_B^3 v^2 \left(\frac{r}{r_B}\right)^{3-2\beta} \sim G\rho^2 r^5 \tag{5}$$

where we have assumed the bullet initially deposits its energy in a volume the size of itself. This implies

$$r_B \propto r^{1+1/\beta} \quad , \quad q = \frac{7\beta+1}{2\beta+1} \quad . \tag{6}$$

The $3/2 < \beta < 3$ condition requires $4/3 < \alpha < 5/3$ and $23/8 < q < 22/7$, both of which are satisfied by all of the impact simulation and dimensional analysis results. Holsapple (1994)

mentions that energy and momentum conservation should represent limiting cases for the impact process and that laboratory experiments involving impacts into sand, rock, and water satisfy those limits. Note that the range in q found in previous studies, $2.95 \leq q \leq 3.11$, spans most of the allowed range for q. This suggests that the catastrophic impact process and α depend on more specific details of the collisions such as the equation of state.

At $r_B \sim r$ there should be no energy loss because the initial energy is deposited in a volume of linear size r. Eq. 5 reflects this. Then the $r_{\rm eq}$ expression in Eq. 3 is still valid.

4. Location of the break

The above calculation of the size spectrum treats $N(r)$ as constant in time. To maintain this steady-state exactly would require the power law to extend to bodies of infinite size, which is impossible. To find the range of masses where this assumption holds, we first find the size $r_{\rm break}$ of the largest KBO to have experienced a destructive collision after an elapsed time τ. We equate the timescale for destructive collisions for each KBO of size $r_{\rm break}$ to τ using Eqs. 1 and 5. To get $N(r)$ we note that bodies of size $r > r_{\rm break}$, having never collided, should be effectively primordial at time τ. For their size spectrum we write $N(r) = N_0 r^{1-q_0}$ where $N_0 \sim 4 \times 10^{7q_0-3}$ cm$^{q_0-1}$ from observations (Trujillo et al. 2001). This is equivalent to a Kuiper belt with 4×10^4 bodies larger than 100 km. They are spread over an area $A \simeq 1200$ AU^2 in the plane of the solar system (Trujillo et al. 2001), so $V \simeq Av/\Omega$ where $\Omega = 0.022$ yr^{-1} is the typical orbital angular velocity of the Kuiper belt. With q for the slope below the break and, as above, q_0 and N_0 for the slope and normalization above the break, we have

$$r_{\rm break} \sim \left[\frac{N_0 \Omega \tau}{A} r_{\rm eq}^{7-2q}\right]^{\frac{1}{4+q_0-2q}} \tag{7}$$

If we set $\tau \simeq 4.5 \times 10^9$ yr to be the age of the solar system, take $3/2 < \beta < 3$, and use the observed $q_0 = 5$, we get

$$20 \text{ km} \lesssim r_{\rm break} \lesssim 50 \text{ km} \quad . \tag{8}$$

This is consistent with the observed break position of $\sim$70 km (Bernstein et al. 2003). Note that if the system had had the high velocity dispersion assumed above over a time considerably shorter than 4.5 Gyr, the break would have occurred at a much smaller KBO size. We therefore infer that the Kuiper belt's current excited state has been a long-lived phase of at least a few billion years' duration rather than a recent phenomenon.

The evolution of the total mass and velocity dispersion of the Kuiper belt is a potential concern, as the break location depends strongly on both. The mass of the Kuiper belt may

have been larger by a factor of $\sim$100 when the solar system was very young ($10^7 - 10^8$ years old) (see, for example, Kenyon 2002). The collision frequency would have been much higher then, so collisions during that period might be expected to have increased the break radius. At that time, though, the velocity dispersion of KBO precursors is believed to have been just $\sim$1 m/s (see, for example, Goldreich et al. 2002). With this impact velocity, $r_{\rm eq} \sim 1$ km, so only targets of size <1 km can be shattered by bullets smaller than they. As a result, collisional evolution during the early solar system should only have affected bodies of size <1 km. The observed break in the spectrum must have been created later. The break location could have been affected if there was a sufficiently long period during which both v and the Kuiper belt mass were large.

With $q \simeq 3$, the mass contained in bodies of size $r \ll r_{break}$ is $N(r)\rho r^3 \propto r$. Since the mass destroyed per unit time is independent of body size, the timescale on which collisional equilibrium is established is $(r/r_{\rm break})\tau \ll \tau$. Then the steady-state approximation—our assumption that the rate at which N changes is much less than the rate of destructive collisions—is self-consistent for $r \ll r_{\rm break}$. Specifically, as $r_{\rm break}$ increases, $N(r_{\rm break})$ decreases—both on a timescale τ—and the $q \simeq 3$ spectrum below $r_{\rm break}$ follows adiabatically (Fig. 2). Our formalism yields the asymptotic spectrum far below $r_{\rm break}$ even though the system is not in steady state overall, since for $r \ll r_{\rm break}$ the destruction rate is faster than the evolution timescale of the system. Dohnanyi (1969) did not discuss the slow decrease in N by which the spectrum differs from a true steady state; he claimed that non-steady-state power-law solutions do not exist. Bernstein et al. (2003) conjectured that the disagreement between their results and Dohnanyi's calculations might indicate a non-steady-state condition in the Kuiper belt. However, the discussion above shows that the fragmentation spectrum below $r_{\rm break}$ should be unaffected by the system's evolution.

As for the lower size boundary, the strength limit derived by Asphaug & Benz (1996) implies that material strength dominates gravity at $r \lesssim 0.3$ km. Impact simulations reach similar conclusions; they put the threshhold in the 0.1–1 km size range (see, for example, Love & Ahrens 1996, Melosh & Ryan 1997, Benz & Asphaug 1999). Below this size threshhold a different q will apply to an equilibrium collisional population. The changes introduced by this effect in the KBO size distribution below $\sim$100 m will affect the size distribution of larger bodies through catastrophic collisions. Numerical simulations of collisional populations indicate that 'waves' may appear in the size spectrum due to a break in the spectrum introduced by a different q (O'Brien & Greenberg 2003). However, the simulations indicate that the average slope of the 'wavy' spectrum is not affected; also, the distribution should asymptotically approach a $q \simeq 3$ spectrum far above this size.

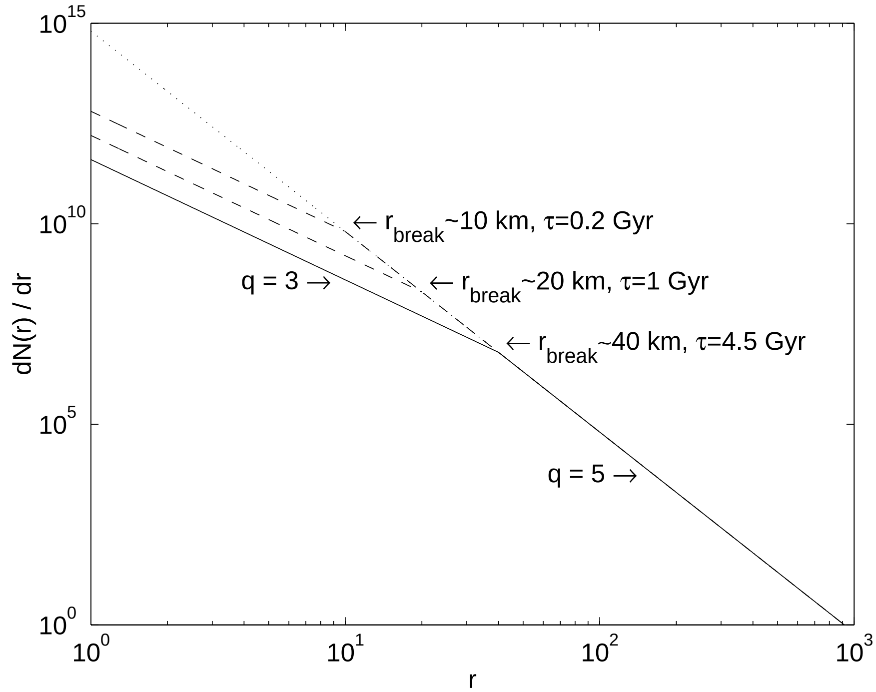

Fig. 2.— Temporal evolution of the number of bodies. Here we use $q = 3$ as a numerical example. The solid line represents the current KBO size distribution. The dotted line is the extrapolation of the large-KBO spectrum to small sizes; we assume this line also represents the primordial size spectrum. Dashed lines show the size spectrum at earlier times $\tau = 0.2$ and 1 billion years. Because $r_{\rm break}$ increases with time, $N(r_{\rm break})$ decreases with time. The evolution of $r_{\rm break}$ and $N(r_{\rm break})$ is much slower than the rates of collisional destruction and creation below $r_{\rm break}$, so these two rates must be very nearly in balance. Then the steady-state approximation is valid in this size range and the spectrum below $r_{\rm break}$ follows a $q = 3$ power law.

5. Summary

We have derived a self-consistent size spectrum $23/8 < q < 22/7$ for a collisional population of bodies whose binding energy is dominated by gravity. We emphasize that this spectrum does not truly represent a steady state; instead, the number density of bodies decreases slowly compared to the collision timescale. For the case of the Kuiper Belt, the spectrum's small-end power-law slope $q \simeq 3$ and break radius $r_{\rm break} \sim 40$ km agree well with those found observationally by Bernstein et al. (2003). Since the power-law slope derived in the steady-state approximation depends heavily on the particular criterion for catastrophic destruction adopted for the bodies, observations of the KBO size spectrum provide a direct

constraint on the bodies' internal structure. The close agreement between this slope and break radius and the best-fit values found by Bernstein et al. (2003) suggests that large KBOs are virtually strengthless bodies held together mainly by gravity. Further surveys of small KBOs between ∼10 and ∼70 km in size would better constrain both the exact position of the actual break in the size spectrum and the power-law slope below the break. Data of this kind would thus confirm or refute our analysis. Such surveys would also allow more detailed comparison of the break locations in the classical and scattered KBO populations, which should reflect differences in the surface densities and velocity dispersions for those two groups.

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